

# A heuristic based Artificial Bee Colony algorithm for Optimum Placement of PMU for complete Observability of Power System

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**Abstract**— This paper presents a novel approach to optimal placement of Phasor Measurement Units (PMUs) for state estimation. At first, an optimal measurement set is determined to achieve full network observability using heuristic approach during normal conditions. An Artificial Bee Colony algorithm is used as an optimization tool to obtain the minimal number of PMUs and their corresponding locations while satisfying associated constraint. The integer based artificial bee colony optimization method and heuristic method are tested on IEEE 14-bus, 30-bus, 57-bus and 118-bus systems.

**Index Terms**— Artificial Bee Colony approach (ABC approach), Heuristic approach, Phasor measurement Unit, Observability, State estimation, Measurement redundancy.

## 1 INTRODUCTION

The wide area monitoring, protection and control of a power system are achievable with the available synchronized measurement technology (SMT). The most significant advantage of this technology are that: (1) the dispersed power system measurement are synchronized through global positioning system (GPS) clock [1]; (2) bus voltage phasor angle can be measured directly; (3) the state estimation speed and accuracy have been improved. The phasor measurement unit (PMU) is an instrument which is developed based on voltage and current phasor of the incident branches linked with the PMU connected node.

PMU placement strategy depends on its purpose of use. For observability optimally located minimum number of PMU is required for every system. Careful study of the result of different existing methods reveals that in every system there a pattern of strategically important bus location. The proposed method searches the pattern of important bus locations by eliminating unnecessary combinations; this reduces the computational burden of this approach. The pattern is searched with help of some heuristic. The pattern is searched computational burden of this approach. The pattern is searched with the help of some heuristics. The heuristic, based on simple network connectivity information are (i) starting PMU installation bus, (ii) the additional bus observability index, (iii) maximum non zero injection bus connectivity,

(iv) "distance criterion" etc. Proposed method systematically selects the bus for PMU placement for every stage using this heuristics. When power system becomes completely observable by a set of PMU, the algorithm stops further processing. Optimality of the set of PMU locations for complete observability of power system. The proposed heuristics based search method consider zero injection measurement as virtual measurement in determining optimal set of PMUs needed to make any system observable. Result based on simulation of a number of test power systems indicate that the proposed method is simple, effective and efficient as compared to the existing methods available in the literature.

This paper is structured as follows; in section 2.1 the problem formulation of PMU placement and redundancy are described. In Section 2.2 The problem formulation of PMU placement based on ABC approach. In Section 3 Case studies were discussed for the optimum location of PMU considering and without considering ZIB for both heuristic approach and ABC approach. for IEEE 14-bus, 30-bus, 57-bus and 118-bus systems.

## 2 PROBLEM FORMULATION

### 2.1 Heuristic Based

For an n-bus system O  $\text{Min} \sum_{i=1}^n w_i x_i$  as follows:

$$\begin{aligned} & \text{Min} \sum_{i=1}^n w_i x_i && \text{Subject to } F(X) \geq b \\ & \text{Subject to } F(X) \geq b && \end{aligned} \quad (1)$$

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Where  $X$  is the binary decision variable vector for PMU placement, whose entries are defined as

$$X_i = 1 \text{ if a PMU is installed at } i^{\text{th}} \text{ bus}$$

$$= 0 \text{ otherwise } (i=1, \dots, N)$$

$b$  is an unit vector of length  $n$ , i.e.  $b = [111 \dots]^T$ .  $w_i$  is the cost of PMU installed at  $i^{\text{th}}$  bus.  $F(X)$  is the observability constraint vector function, whose entries are nonzero if the corresponding individual buses are observable w.r.t a given measurement set and zero otherwise. If  $w_i$  is constant, the OPMP is represented as follows

$$\text{Min } \sum_{i=1}^n x_i \tag{2}$$

Subject to  $F(x) \geq b$

Constraint vector function ensures full network observability. A solution i.e. set of minimum  $x_i$  is to be found out which will satisfy (2). The constraint vector function is formed using the binary connectivity matrix ( $A$ ) of power system. The binary connectivity matrix ( $A$ ) represents the bus connectivity information of a power system. The elements of matrix  $A$  for bus  $m$  and bus  $n$  is defined as

$$A_{m,n} = 1 \text{ if } m = n$$

$$= 1 \text{ if bus } m \text{ is connected to bus } n$$

$$= 0 \text{ otherwise}$$

The constraint vector function for any particular test system is in (3), (4). If any  $x_i$  appearing in  $f_j$  in  $F$  non zero the system will be completely observable.

$$F(X) = AX \geq b \tag{3}$$

$$f_i = a_{i1}x_1 + \dots + a_{ii}x_i + \dots + a_{iN}x_N \tag{4}$$

2.1.1 Algorithm for Heuristic approach

For observability optimally located minimum number of PMU is required, in the result of the exhaustive approach and other existing method, it is found that most of the bus locations are common in different optimal solution for any particular power system. These are the strategically important bus location for that system. In every power system there is a pattern of strategically important bus locations. Once this pattern is identified, the PMU placement becomes an easy job. In the proposed approach the pattern is being investigated and obtained using the following heuristics.

1) *Starting Bus for Installation*: Buses connected to radial bus are selected a starting bus for PMU placement. Reasons for this are clearly mentioned in [7] such as i) A PMU along with one additional bus with which it is connected. ii) A PMU placed at the bus connected to radial bus will make more than one bus observable along with the radial bus. But in such case a ZIB connected to radial bus is not selected because it is a free meas-

urement and applying KCL at ZIB along with its other connected observable buses the radial bus can be made IOB.

2) *Additional Bus Observability Index (ABOI)*: It indicates the number of additional buses that will become observable if a PMU is installed at any particular Bus. It includes the installed bus also, unless it is already observable. The bus having maximum value of ABOI is selected for PMU placement bus. The reason is that the deployment of PMU at any bus should make maximum number of bus observable

3) *Criterion* ( $\alpha > 0$ ):  $\alpha_i = \max^m$  (ABOI) - number of PMU bus connected to  $i^{\text{th}}$  bus. In case few bus have some ABOI the bus with minimum PMU bus connection is selected as PMU placement bus. It will help in pruning operation.

4) *Maximum number of Load or generation bus connectivity bus*: This indicates the number of load or generation bus (a non-zero injection bus) connected to each bus including the bus itself (if it is a load or generation bus). This is evaluated as number of bus connectivity of any bus minus the number of zero injection bus connected to that bus. The reason is that the ZIB are observable, KCL provides the voltage phasor of ZIB.

5) *Distance criterion*: Let bus  $K$ ,  $L$  and  $M$  are already PMU installed bus. For any competitive  $i^{\text{th}}$  bus find 'SUMDIST' USING (6). Bus corresponding to maximum value of 'SUMDIST' is the choice of PMU bus. This will help in finding most distant bus for selecting PMU bus and this will avoid getting trapped in local minimum.

$$\text{SUMDIST}(i) = \sum_{t=K,L,M} \text{Bus\_Dist}(i,t) \tag{5}$$

TABLE 1  
 OPTIMAL PMU PLACEMENT AND MEASUREMENT REDUNDANCY FOR IEEE 14 BUS SYSTEM

PMU Placement	Number of buses													
	1	2	3	4	5	6	7	8	9	10	11	12	13	14
2,6,7,9	1	1	1	3	2	1	2	1	2	1	1	1	1	1
2,7,10,13	1	1	1	2	1	1	1	1	2	1	1	1	1	1
2,7,11,13	1	1	1	2	1	2	1	1	1	1	1	1	1	1

If a bus is observed twice by a PMU then redundancy value of that bus is increased by one. For example, the optimal PMU location for the network observability of IEEE 14 bus system is listed in table 1. The three PMU sets satisfy the observability criteria, but the measurement redundancy value will be varied. The number of bus traced more than once are 4, 2 and 2 by the optimal PMU set 1, 2 and 3 respectively.

2.2 Artificial Bee Colony Algorithm for Optimal Placement of Phasor Measurement Units

2.2.1 ILLUSTRATIONS

(a) System with no conventional measurements and/or measurements.

In this case the flow measurement and the zero injection are ignored. In order to form the constraint set, binary connectivity

matrix A whose entries are defined below. Matrix A can be directly formed from the bus admittance matrix by transforming its entries into binary form.

$$A_{m,n} = 1 \text{ if } m = n \tag{6}$$

$$= 1 \text{ if bus } m \text{ is connected to bus } n$$

$$= 0 \text{ otherwise}$$

The constraints for this case is formulated as

$$F(X) = AX \tag{7}$$

The

$$f_i = A(i,:) * X(i,1) \text{ i-buses of the grid} \tag{8}$$

$$f_1 = (X_1 + X_2 + X_5) \geq 1$$

$$f_2 = (X_1 + X_2 + X_3 + X_4 + X_5) \geq 1 \tag{9}$$

The operator + serves logical OR, and the use of 1 in R.H.S ensures atleast one of the variable appearing in the sum will be non zero which means one of atleast 1,2 and 5 numbered buses of eqn (10) should be provided with a PMU to make bus1 observable.

(b) System with atleast one flow measurements or zero injections:

This case considers the situation where some flow measurements may be present. Existence of flow measurements will lead to the modification of constraints for buses accordingly. Modification follows the observation tha having a flow measurement along a given branch allows the calculation of one of the terminal bus voltage phasors when the other one is known. Hence the constraint associated with the terminal buses of the measured branch can be merged into a single constraint. In the case of the example system, the constraint for buses 5 & 6 are merged into a joint constraint as there is flow measurement in branch between 5 & 6.

$$f_5 = (X_1 + X_2 + X_4 + X_5 + X_6) \geq 1$$

$$f_6 = (X_{11} + X_{12} + X_{13} + X_5 + X_6) \geq 1 \tag{10}$$

$$f_{5-6_{new}} = f_5 + f_6 \tag{11}$$

$$= (X_1 + X_2 + X_4 + X_5 + X_6 + X_{11} + X_{12} + X_{13}) \geq 1$$

(C) System with flow measurements and zero injections.

This case considers the most general situation where both injection and flow measurements may be present, but not enough to make the entire system observable. Injection measurements whether they are zero injections or not, are treated the same way. Consider again the 14 bus system shown in fig (1), where bus 7 is a zero injection bus. It is easy to see that if the phasor voltages at any three out of the four buses 4, 7, 8 and 9 are known, then the fourth one can be calculated using Kirchoff's Current Law at bus 7 where the net injection current is known.

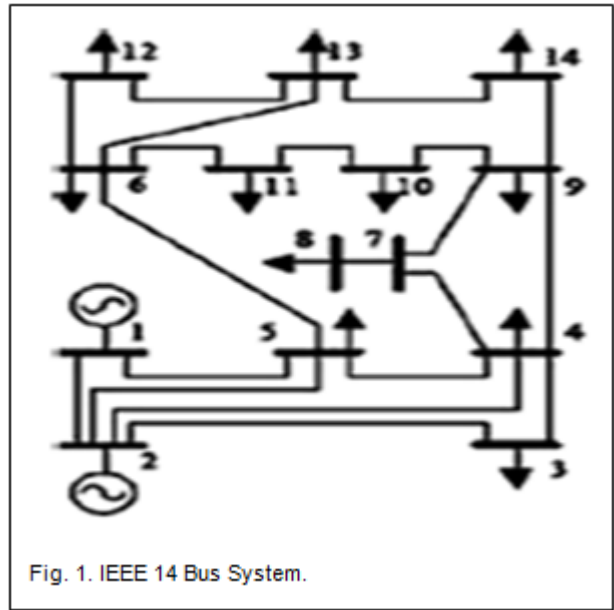


Fig. 1. IEEE 14 Bus System.

This is accomplished as shown below. To treat the zero injection bus 7 in the IEEE 14-bus system, constraints associate with its neighboring buses 4, 8, and 9 will be modified as follows

$$f_4 = (X_2 + X_3 + X_4 + X_5 + X_7 + X_9 + f_7 \cdot f_8 \cdot f_9) \geq 1$$

$$f_7 = (X_4 + X_7 + X_8 + X_9 + f_4 \cdot f_8 \cdot f_9) \geq 1 \tag{12}$$

$$f_8 = (X_7 + X_8 + f_4 \cdot f_7 \cdot f_9) \geq 1$$

$$f_9 = (X_4 + X_7 + X_9 + X_{10} + f_4 \cdot f_7 \cdot f_8) \geq 1$$

The operator '.' in the above equations serves as the logical "AND" operation. The expressions for fi can be further simplified by using the following properties of the Boolean logical AND (.) and OR (+) operators.

By substituting  $f_7, f_8$  and  $f_9$  in expression for  $f_4$  can be written as

$$f_4 = (X_2 + X_3 + X_4 + X_5 + X_7 + X_9 + X_8 \cdot X_{10} + X_3 \cdot X_{14}) \tag{13}$$

Applying similar simplification to other expressions, other constraints can be redefined as

$$f_5 = (X_4 + X_7 + X_8 + X_9)$$

$$f_9 = (X_4 + X_7 + X_9 + X_{14} + X_2 \cdot X_3 + X_3 \cdot X_8 + X_3 \cdot X_9) \tag{14}$$

### 2.2.2. Artificial Bee Colony algorithm for optimal placement of PMU

1. Generate n random solutions with in boundaries of the system
  - a. X=Boolean (rand (No of solutions, size of solutions))
2. Check that random solutions satisfy the inequality constraints of buses ie
  - a.  $f(X) = (A \cdot X) \geq 1$
  - b. where A is binary impedance matrix, X is the solution.
3. Calculate the objective function and fitness of each solution
4. Store the best fit as Xbest solution
5. A mutant solution is formed using a randomly selected neighbor
  - a. If (rand>0.5)

- b.  $X_{kmutant} = X_k(i) \text{ OR } X_j(i)$
- c. Else
- d.  $X_{kmutant} = X_k(i) \text{ AND } X_j(i)$
- e. where  $j$  is the randomly selected neighbor and  $i$  is a random parameter.
- 6. Check the constraints  $f(X) = (A.X) \geq 1$ . If the constraints are satisfied proceed to step 7, else move to step 5
- 7. Replace  $X_{kmutant}$  by  $X_k$ , if the mutant has higher fitness or lowest cost of PMU.
- 8. Repeat the above procedure for all solutions.
- 9. Onlooker bee phase (Simple ABC)
- 10. Probability of each solution is calculated
  - a. Probability (i) =  $a \cdot \text{fitness}(i) / \max(\text{fitness}) + b$
  - b. where  $\{a+b=1\}$
- 11. The solution  $X_{best}$  is selected if its probability is greater than a random number.
  - If ( $\text{rand} < \text{probability}(i)$ )
  - Solution is accepted for mutation
  - Else
  - Solution is discarded for mutation.
- 12. Again the best  $X_{best}$  is determined
- 13. Replace a  $X$  by random  $X$  if its trial counter exceed threshold (Scout bee phase)
- 14. Repeat the above for max no of iterations.
- 15. The  $X_{best}$  &  $f(x_{best})$  are the best solution and Global minimum of the objective function.

### 3 CASE STUDIES AND DISCUSSION ON SIMULATION RESULTS

The proposed two stage algorithm determines the minimum number of strategic bus locations where PMU must be placed for complete observability. First stage of the algorithm determines the important bus locations for allocating PMUs. The second stage is pruning stage checks the possible ways to further reduce any PMUs. Minimum number of strategic bus locations where PMU must be placed for complete observability. The integer based artificial bee colony optimization method is tested on IEEE 14-bus, 30-bus, 57-bus and 118-bus systems. The optimum locations for both heuristic and Artificial Bee colony approach were compared for IEEE standard test systems. Tables given below show the optimum location of PMU considering and without considering ZIB for both heuristic approach and ABC approach. For IEEE 14-bus, 30-bus, 57-bus and 118-bus systems.

TABLE 2  
 OPTIMAL PMU PLACEMENT RESULTS FOR NORMAL OPERATING CONDITION WITHOUT CONSIDERING ZIB

System	Optimal PMU location	No of PMU
IEEE 14 bus system	2,6,7,9,4	4
IEEE 30 bus system	9,12,25	10
IEEE 57 bus system	1,4,9,20,24,27,29,30,32,36,38,39,41,45,46,51,54,17	17
IEEE 118 bus system	1,5,9,12,13,17,21,23,26,28,34,32,37,41,45,49,53,56,62,63,68,71,75,77,80,85,86,90,94,101,105,110,114,32	32

TABLE 3  
 OPTIMAL PLACEMENT RESULTS FOR NORMAL OPERATING CONDITION WITH CONSIDERING ZIB

System	ZIB Locations	Optimal PMU Location	No of PMU
IEEE14	7	2,6,9,3	4
IEEE30	6,9,22,25,27,28	2,3,10,12,18,24,30	7
IEEE 57	4,7,11,21,22,24,26,34,36,37,39,40,45,46,48	1,6,13,19,25,29,32,38,51,54,56	11
IEEE 118	5,9,30,37,38,63,64,68,71,81	1,6,8,12,15,17,21,25,29,34,40,45,49,53,56,62,72,75,77,80,85,86,90,94,101,105,110,114	28

**TABLE 4**  
OPTIMAL PMU PLACEMENT RESULTS FOR NORMAL OPERATING  
CONDITION WITHOUT CONSIDERING ZIB

System of PMU	Optimal PMU location	No
IEEE 14 bus system	2,7,11,13	4
IEEE 30 bus system	1,5,9,10,12,15,18,25,27,28	10
IEEE 57 bus system	1,6,13,15,18,21,22,25,27,29 32,34,38,40,41,46,51,54,57	19
IEEE 118 bus system	2,5,9,11,12,17,21,24,25,28, 34,37,40,45,49,52,56,62,63 68,73,75,77,80,85,86,90,94 101,105,110,114	32

**TABLE 5**  
OPTIMAL PMU PLACEMENT RESULTS FOR NORMAL OPERATING  
CONDITION WITH CONSIDERING ZIB

System	Optimal PMU location	No of PMU
IEEE 14	2,6,9	3
IEEE 30	3,5,10,12,18,23,27	7
IEEE 57	1,6,9,15,20,25,27,32 38,47,50,53,56	13
IEEE 118	2,8,11,12,15,19,21,27,31, 32,34,40,45,49,56,62,65 72,75,77,80,85,86,90,94, 101,105,110	29

\*ZIB loctions is as mentioned in Table 3

**TABLE 6**  
MAXIMUM NUMBER OF PMU TO MAKE THE SSTEM OBSERVABLE  
UNDER NORMAL OPERATING CONDITION

Test system	Heuristic Approach	Artificial Bee colony Approach
IEEE 14-bus	4	3
IEEE 30-bus	7	7
IEEE 57-bus	11	15
IEEE 118-bus	28	29

## 4 CONCLUSION

A new methodology for optimal placement of PMU is presented in this paper. The new iterative method makes the tests systems topological observable by placing a set of minimum PMUs. The three stage algorithm is simple, fast and easy to implement. The results of this heuristic method are compared with Artificial Bee Colony approach. The present method obtains optimal solution using simple network connectivity information. The overall optimal solution obtained is sufficient to take care of system observability under normal operating condition. Measurement redundancy is also checked. Simulation results for different networks show the effectiveness of the proposed method in obtaining the minimum number of PMU required for complete observability of power systems. Drawback of heuristic is execution time and it is mitigated using Artificial Bee Colony approach.

## 5 REFERENCES

- [1] A.G.Phadke, "Synchronized phasor measurements in po-wer sys tems". IEEE Comput.Applicat. Power ,vol.6,no.2,pp.10-15,Apr.1993.
- [2] T. L. Baldwin, L. Mili, M. B. Boisen, and R. Adapa, "Power system observability with minimal phasor measurement placement," *IEEE Trans.Power Syst.*, vol. 8, no. 2, pp. 701–715, May 1993.
- [3] Xu B, Abur A. Observability analysis and measurement placement for systems with PMUs. In: Proceedings of the IEEE PES power systems conference and exposition 2004;2: p. 943–6.
- [4] R. F. Nuqui and A. G. Phadke, "Phasor measurement unit placement techniques for complete and incomplete observability," *IEEE Trans.Power Del.*, vol. 20, no. 4, pp. 2381–2388, Oct. 2005.
- [5] J. Chen and A. Abur, "Placement of PMUs to enable bad data detection in state estimation," *IEEE Trans. Power Syst.*, vol. 21, no. 4, pp.1608–1615, Nov. 2006.
- [6] B. Milosevic and M. Begovic, "Non-dominated sorting genetic algorithm for optimal phasor measurement placement," *IEEE Trans. Power Syst.*, vol. 18, no. 1, pp. 69–75, Feb. 2003.
- [7] J. Chen and A. Abur, "Placement of PMUs to enable bad data detection in state estimation," *IEEE Trans. Power Syst.*, vol. 21, no. 4, pp.1608–1615, Nov. 2006.
- [8] B. Gou, "Optimal placement of PMUs by integer linear programming," *IEEE Trans. Power Syst.*, vol. 23, no. 3, pp. 1525–1526, Aug. 2008.
- [9] C. Madtharad, S. Premrudeepreechacharn, N. R.Watson, and R. Saeng-Udom, "An optimal measurement placement method for power system harmonic state estimation," *IEEE Trans. Power Del.*, vol. 20, no. 2, pt.2, pp. 1514–1521, Apr. 2005.
- [10] I. Kamwa, R. Grondin, "PMU configuration for system dynamic performance measurement in large multiarea power systems," *IEEE Transactions on Power Systems*, Vol. 17, No. 2May 2002, pp. 385-394.
- [11] D.Karaboga, "An idea Based on Honey Bee Swarm for Numerical Optimization", Technical report-Tr06t, Erciyes University, Engineering.
- [12] A. Abur and F. H. Magnago, "Optimal meter placement for maintaining observability during single branch outage," *IEEE Trans. Power Syst.*, vol. 14, no. 4, pp. 1273–1278, Nov. 1999.
- [13] J. Peng, Y. Sun, H. Wang, Optimal PMU placement for full network observability using Tabu search algorithm, *International Journal of Electrical Power & Energy Systems* 28 (4) (2006) 223–231.
- [14] F. Marin, F. Garcia-Lagos, G. Joya, F. Sandoval, Genetic algorithms for optimal placement of phasor measurement units in electrical net-

- works, Electronics Letters 39 (19) (2003) 1403–1405.
- [15] D. Simon, Biogeography-based optimization, IEEE Transactions on Evolutionary Computation 12 (6) (2008) 702–713.
- [16] S. Chakrabarti, E. Kyriakides, Optimal placement of phasor measurement units for power system observability, IEEE Transactions on Power Systems 23 (3) (2008) 1433–1440.
- [17] Christie R. Power system test archive; August 1999. <<http://www.ee.washington.edu/research/pstca>>.
- [18] Pai MA. Energy function analysis for power system stability. Norwell: Kluwer Academic Publishers; 1989.

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